USN | | | | | | | | | | 06MAT31

Third Semester B.E. Degree Examination, December 2010
Engineering Mathematics – III

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Find the Fourier series for the function $f(x) = x(2\pi - x)$ over the interval $(0, 2\pi)$ and deduce that $\frac{\pi^2}{12} = \sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$. **(07 Marks)**

b. Obtain the half-range sine series for
$$f(x) = \begin{cases} \frac{1}{4} - x, & \text{for } 0 < x < \frac{1}{2} \\ x - \frac{3}{4}, & \text{for } \frac{1}{2} < x < 1 \end{cases}.$$ **(07 Marks)**

c. Obtain the constant term and the co-efficients of sin θ and sin 2 θ in the Fourier expansion of y given the following data **(06 Marks)**

θ°	0	60	120	180	240	300	360
y	0	9.2	14.4	17.8	17.3	11.7	0

2 a. Obtain the finite Fourier sine transform of the function $f(x) = \cos kx$, where k is a non integer, over $(0, \pi)$. **(07 Marks)**

b. Find the Fourier sine and cosine transforms of $f(x) = e^{-\alpha x}$, $\alpha > 0$. **(07 Marks)**

c. Find the inverse Fourier transform of e^{-u^2}. **(06 Marks)**

3 a. Form the partial differential equation by eliminating the arbitrary functions from $Z = f(x + I t) + g(x - i t)$, where $i = \sqrt{-1}$. **(07 Marks)**

b. Solve by the method of separation of variables $p y^3 + q x^3 = 0$. **(07 Marks)**

c. Solve $(mz - ny) p + (nx - lz)q = ly - mx$. **(06 Marks)**

4 a. Derive the one – dimensional heat equation. **(07 Marks)**

b. Obtain the D'Almbert's solution of the wave equation $u_{tt} = c^2 u_{xx}$, subject to the condition $u(x, 0) = f(x)$ and $\frac{\partial u}{\partial t}(x,0) = 0$. **(07 Marks)**

c. Solve the wave equation $c^2\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 < x < \pi$, given $u(0, t) = u(\pi, t) = 0$; $u(x, 0) = 0$; $\frac{\partial u}{\partial t}(x, 0) = A(\sin x - \sin 2x)$, $A \neq 0$. **(06 Marks)**

PART – B

5 a. Find the smallest and the largest roots of $e^x - 4x = 0$, correct to 4 decimal places by Newton – Raphson method. **(07 Marks)**

b. Solve by Gauss elimination method
$2x_1 + x_2 + 4x_3 = 12$; $4x_1 + 11x_2 - x_3 = 33$; $8x_1 - 3x_2 + 2x_3 = 20$. **(07 Marks)**

c. Find the largest eigenvalue and the corresponding eigenvector of the matrix by using power method :
$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$ taking $[1, 1, 1]^T$ as the initial eigenvector, perform 5 iterations. **(06 Marks)**

6 a. Using the Lagrange' formula, find the interpolating polynomial that approximates to the function described by the following table : **(07 Marks)**

X	0	1	2	3	4
f(x)	3	6	11	18	27

Hence find f(0.5) and f(3.1)

b. A rod is rotating in a plane. The following table gives the angle θ (in radians) through which the rod has turned for various values of t (in seconds)

t	0	0.2	0.4	0.6	0.8	1.0	1.2
θ	0	0.12	0.49	1.12	2.02	3.20	4.67

Calculate the angular velocity and angular acceleration of the rod at $t = 0.4$ second. **(07 Marks)**

c. Evaluate $\int_0^1 \frac{x}{1+x^2}dx$ by using the Simpson's (⅜)th rule, dividing the interval into 3 equal parts. Hence find an approximate value of $\log\sqrt{2}$. **(06 Marks)**

7 a. Derive the Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$. **(07 Marks)**

b. Solve the variational problem :

$\delta \int_0^1 \left(x + y + y'^2\right)dx = 0$ under the conditions $y(0) = 1$ and $y(1) = 2$. **(07 Marks)**

c. Find the geodesics on a surface given that the arc length on the surface is

$S = \int_{x_1}^{x_2} \sqrt{x(1+y'^2)}\, dx$. **(06 Marks)**

8 a. Find the Z-transform of

i) $3n - 4\sin\frac{n\pi}{4} - 5a^2$

ii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$. **(07 Marks)**

b. Obtain the inverse Z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. **(07 Marks)**

c. Solve the difference equation $u_{n+2} - 5u_{n+1} + 6u_n = 2$, with $u_0 = 3$, $u_1 = 7$ using z-transforms. **(06 Marks)**

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Third Semester B.E. Degree Examination, December 2010
Analog Electronic Circuits

Time: 3 hrs. Max. Marks:100

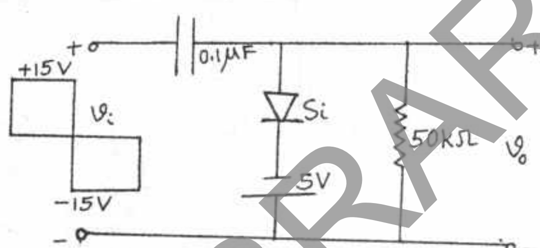
Note: ***Answer any FIVE full questions, selecting atleast TWO Questions from each of Part – A and Part - B.***

PART - A

1 a. What is an equivalent circuit of a device? Explain the different equivalent circuits for semiconductor diode. **(07 Marks)**

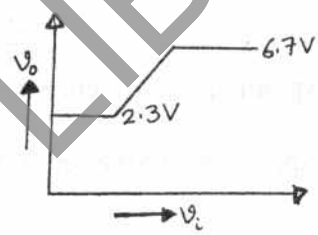
b. Analyse the circuit shown below, Fig. Q1(b), and draw the output waveform. Assume $V_r = 0.7V$. **(08 Marks)**

Fig. Q1(b)



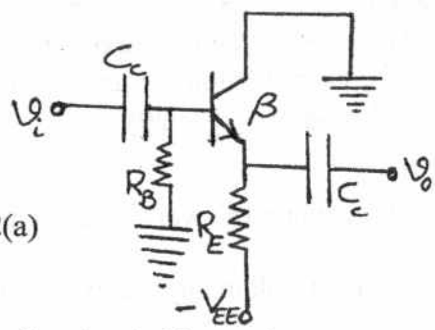
c. Write a suitable circuit to get the following transfer characteristics fig. Q1(c), and explain its working. **(05 Marks)**

Fig. Q1(c)



2 a. Find I_{C_Q} and V_{CE_Q} for the circuit shown, Fig. Q2(a). **(05 Marks)**

Fig. Q2(a)



b. Find the coordinates of the Q point and locate it on the dc load line for the voltage divider configuration. Given $V_{CC} = 16V$, R_1 (upper resistor) = 62 kΩ , $R_2 = 9.1$ kΩ , $R_C = 3.9$kΩ , $R_E = 0.68$kΩ and $\beta = 80$. The coupling capacitors are 10μF each. Also find V_C , V_E and V_B. **(08 Marks)**

c. Define turn ON time and turn OFF time of a transistor. Design a transistor inverter if $V_{CC} = 10V$, $I_{C_{sat}} = 10mA$ and $\beta = 250$. Assume input to be a pulse of amplitude 10V. **(07 Marks)**

3 a. What is bias stabilization? Explain. Derive an expression for $S(I_{CO})$ and $S(V_{BE})$ for fixed bias configuration. **(08 Marks)**

b. For an emitter bias circuit (capacitor is bypassed), determine r_e, Z_i, Z_o and A_v. Given $R_B = 470\ k\Omega$, $R_C = 2.2k\Omega$, $V_{CC} = 20V$, $R_E = 0.56k\Omega$, $C_E = 10\mu F$, $\beta = 120$, $r_o = 40k\Omega$, $C_C = 10\mu F$. **(08 Marks)**

c. Determine i) the common logarithm of the number 2.2×10^3. ii) the power gain in decibels for $P_o = 100$ m watts, $P_i = 5$m watts. **(04 Marks)**

4 a. The transistor is connected as a CE amplifier. Determine Z_c, Z_o, A_I and A_V using complete hybrid model. **(10 Marks)**

b. Discuss the low frequency and high frequency response of a RC coupled amplifier. **(10 Marks)**

PART - B

5 a. For the circuit of fig. Q5(a), calculate the dc bias voltage V_E. **(05 Marks)**

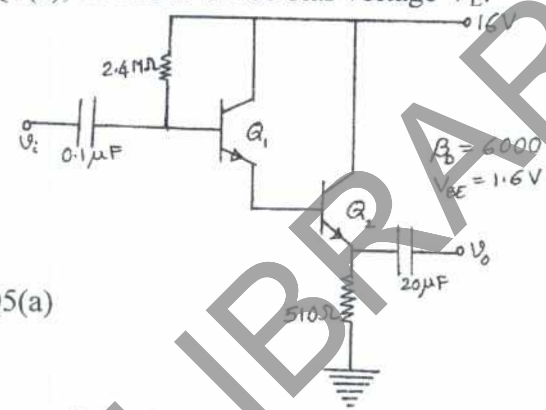


Fig. Q5(a)

b. With a block diagram, explain the difference between voltage series and voltage shunt feedback. **(05 Marks)**

c. Using the block diagram approach, derive an expression for A_f and Z_{of} for current series feedback amplifier. **(10 Marks)**

6 a. With a neat circuit diagram, explain the operation of a transformer coupled class A power amplifier. **(10 Marks)**

b. For a class B amplifier with $V_{CC} = 25V$ driving an 8Ω load, determine i) maximum I/P power ii) maximum O/P power iii) maximum circuit efficiency. **(06 Marks)**

c. Calculate the second harmonic distortion for an O/P waveform having measured values of $V_{CE_{min}} = 2.4V$, $V_{CE_Q} = 10V$ and $V_{CE_{max}} = 20V$. **(04 Marks)**

7 a. With a neat circuit diagram, explain the working principle of RC phase shift oscillator, with relevant equations. **(10 Marks)**

b. What are the tuned oscillators? Explain any one type of tuned oscillator. **(10 Marks)**

8 a. Define g_m and r_d of field effect transistor. Explain the procedure to determine the above values graphically. **(06 Marks)**

b. Write the ac equivalent circuit for voltage divider JFET configuration and determine Z_i, Z_o and A_v. **(10 Marks)**

c. Differentiate between enhancement and depletion MOSFET. **(04 Marks)**

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Third Semester B.E. Degree Examination, December 2010
Logic Design

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the definition of combinational logic. **(04 Marks)**

b. Simplify the function $y = f(a, b, c, d) = \Sigma\, m(2,3,4,5,13,15) + \Sigma\, d(8,9,10,11)$, using Karnaugh map. **(05 Marks)**

c. Simplify the function $y = f(a,b,c,d) = \pi M(0,4,5,7,8,9,11,12,13,15)$ using the Karnaugh map. **(05 Marks)**

d. Simplify the function $y = f(a,b,c,d,e) = \Sigma\, m(0,2,8,10,16,18,24,26)$ using the Karnaugh map. **(06 Marks)**

2 a. Simplify using the Quine-McClusky minimization technique $y = f(a,b,c,d) = \Sigma\, m(0,2,8,10)$. **(10 Marks)**

b. Simplify using the variable entered method (VEM),
$y = f(a,b,c,d,e) = \Sigma\, m(1,3,4,6,9,11,12,14,17,19,20,22,25,27,28,30) + \Sigma\, d(8,10,24,26)$. **(10 Marks)**

3 a. Perform the following to design a combinational logic circuit to convert the BCD digit to an excess – 3 BCD digit.
i) Construct the truth table, ii) write the min term equation for each output function, iii) Simplify the output function and write reduced logic equation and iv) Draw the resulting logic diagram. **(10 Marks)**

b. Draw the logic diagram for 2-to-4 logic decoder, with an active low encoder enable and active high data outputs. Construct a truth table and identify the data inputs, the enable input and the outputs. Describe the circuit function. Draw the logic symbol for decoder. **(10 Marks)**

4 a. Perform the following to design a full-subtractor:
i) Construct the truth table and simplify the output equations. ii) Draw the resulting logic diagram and iii) Realize the subtractor, using a decoder. **(10 Marks)**

b. Write a true table for two-bit magnitude comparator. Write the Karnaugh map for each output of two-bit magnitude comparator and the resulting equation. **(10 Marks)**

PART – B

5 a. What is the difference between a flip-flop and a latch? What is the gated SR latch? Explain the operation of gated SR latch, with a logic diagram, truth table and logic symbol. **(10 Marks)**

b. Explain the operation of positive-edge-triggered JK flip-flop and T flip-flop, with the help of logic diagram, function table and logic symbol. **(10 Marks)**

6 a. Explain the working principle of four-bit binary ripple counter, with the help of a logic diagram, timing diagram and counting sequence. **(10 Marks)**

b. Explain the design of a synchronous Mod-6 counter, using the clocked flip-flops. Clearly indicate the application table, excitation table and minimal sum expressions. **(10 Marks)**

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

7 a. Draw the Mealy and Moore synchronous machine models. Label the excitation variables, state variables, input variables and output variables, in both the diagrams. **(08 Marks)**

b. For the logic diagram given in Fig.Q7(b),
   i) Derive the excitation and output equations,
   ii) Write the next state equations,
   iii) Construct a transition table and
   iv) Draw the state diagram. **(12 Marks)**

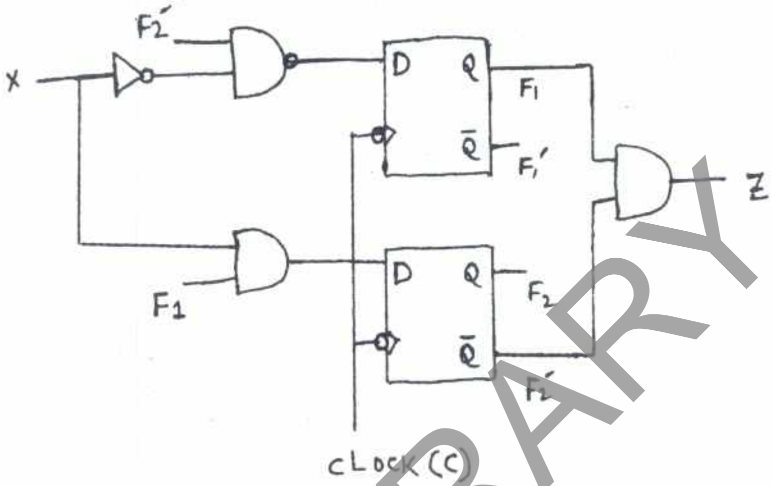

Fig.Q7(b)

8 Design a cyclic module-8 synchronous binary counter, using JK flip-flops, to count the number of occurrences of an input, i.e., the number of times it is a 1. The input variable x must be coincident with the clock to be counted. The counter is to count in binary. The design should clearly indicate the following :

a. State diagram and state table. **(05 Marks)**
b. Transition table and excitation table. **(05 Marks)**
c. Karnaugh maps **(05 Marks)**
d. Logic diagram. **(05 Marks)**

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06ES34

Third Semester B.E. Degree Examination, December 2010
Network Analysis

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Standard notations are used.
3. Missing data may be suitably assumed.

PART – A

1 a. Use the node analysis and find the value of V_x in the circuit shown in Fig.Q1(a), such that the current through the impedance $(2 + j3)\Omega$ is zero. **(10 Marks)**

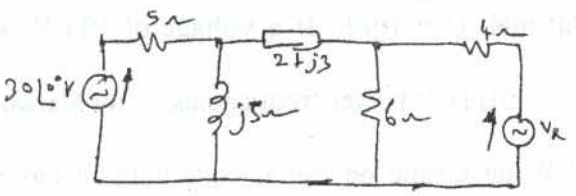


Fig.Q1(a)

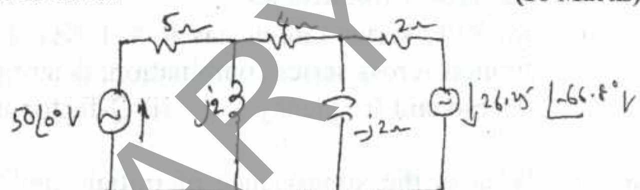
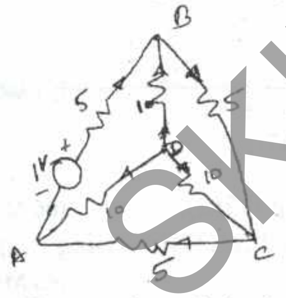


Fig.Q1(b)

b. Find the current through the 4Ω resistor, using the loop current method, in the circuit shown in Fg.Q1(b). **(10 Marks)**

2 a. Explain the following terms with reference to network topology :
i) Tree ii) Branch iii) Cut set matrix iv) Tie set matrix **(08 Marks)**

b. For the given resistance network, write a tie set schedule and obtain equilibrium equations on current basis. Calculate the values of branch voltages. Assume inner branches as tree branches. [Refer Fig.Q2(b)] **(12 Marks)**



All resistances in ohms

Fig.Q2(b)

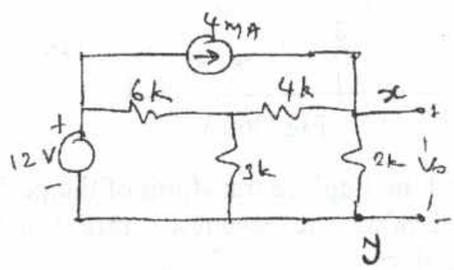


Fig.Q3(b)

3 a. State and explain the Thevenin's theorem. **(06 Marks)**

b. Obtain the Thevenin's equivalent of the network shown in Fig.Q3(b) between the terminals x and y. Also find V_o. **(06 Marks)**

c. Using Worton's theorem, find the current through the load impedance Zl in Fig.Q3(c). **(08 Marks)**

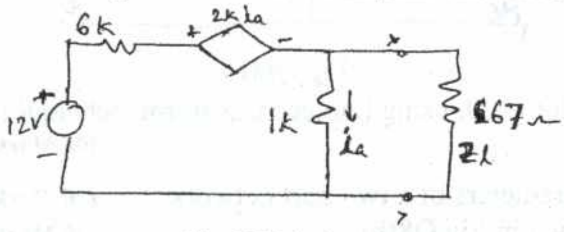


Fig.Q3(c)

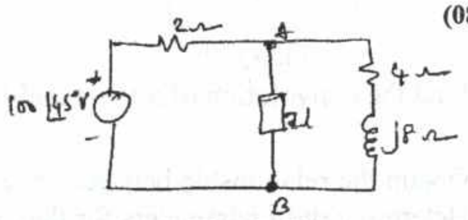


Fig.Q4(b)

4 a. Derive the condition or maximum power transfer across a load from a source as applied to a simple A.C. circuit. Assume a suitable circuit for your mathematical derivation. **(08 Marks)**

4 b. Find the load impedance to be connected across the terminals AB for the maximum power transfer. The network is shown in Fig.Q4(b). Also find the maximum power delivered to the load. **(06 Marks)**

c. Write a note on superposition theorem as applied to a dc circuit. **(06 Marks)**

PART – B

5 a. Show that a two branch parallel resonant circuit is resonant at all the frequency, if $R_l = R_C = \sqrt{\dfrac{L}{C}}$ where R_l = resistance in the inductor branch, R_C = capacitor in the capacitor branch. **(08 Marks)**

b. Derive for a resonant circuit, the resonant frequency $f_o = \sqrt{f_1 f_2}$, where, f_1 and f_2 are the two half power frequencies. **(08 Marks)**

c. An RLC series circuit has R = 1 KΩ, L = 100 mH, C = 10μF. If a voltage of 100 V is applied across series combination, determine :
i) Resonant frequency ii) Q-factor and iii) Half power frequencies. **(04 Marks)**

6 a. What is the significance of initial conditions? Write a note on initial conditions in basic circuit elements. **(04 Marks)**

b. How is time constant of an RL circuit is defined? Explain its importance in transient analysis, with a suitable example. **(08 Marks)**

c. In the circuit shown in Fig.Q6(c), the switch K is changed from position 1 to position 2 at t = 0, the steady state having been reached before switching. Find the values of i, $\dfrac{di}{dt}$ and $\dfrac{d^2i}{dt^2}$ at t = 0. **(08 Marks)**

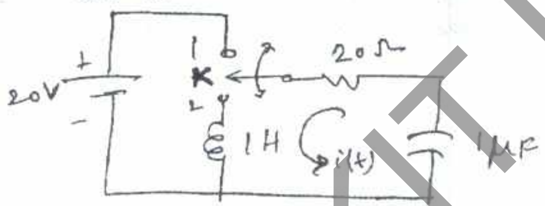


Fig.Q6(c)

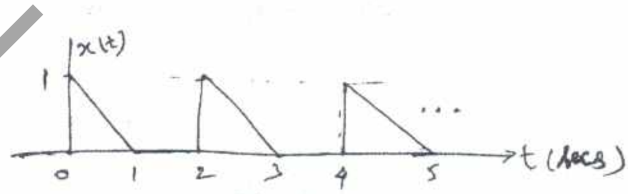


Fig.Q7(a)

7 a. Find the Laplace transform of the periodic signal x(t) shown in Fig.Q7(a). **(06 Marks)**

b. Determine the response current i(t) in the circuit shown in Fig.Q7(b) using Laplace transform. **(06 Marks)**

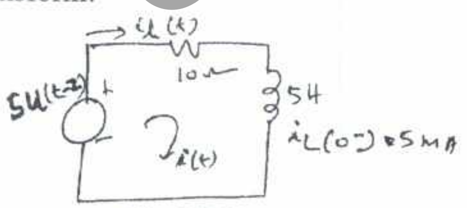


Fig.Q7(b)

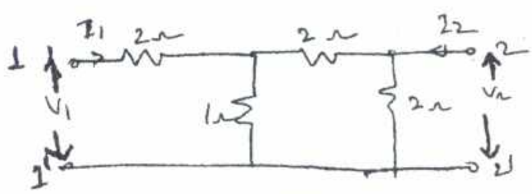


Fig.Q8(b)

c. Find the convolution of h(t) = t and f(t) = $e^{-\alpha t}$ for t > 0, using Laplace transform techniques. **(08 Marks)**

8 a. Obtain the relationship between 'h' and 'y' parameters of a two port network. **(10 Marks)**

b. Determine the z parameters for the circuit shown in Fig.Q8(b). **(06 Marks)**

c. Write a note on ABCD parameters. **(04 Marks)**

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06ES36

Third Semester B.E. Degree Examination, December 2010
Field Theory

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting at least TWO questions from each part.***
2. Assume any missing data suitably.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Show that the electric field intensity at a point, due to 'n' number of point charges, is given by $\overline{E} = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{Q_i}{R_i^2}\hat{a}_{R_i}\ v/m$. **(05 Marks)**

b. A uniform line charge of infinite length with $\rho_L = 40$ nc/m , lies along the z-axis. Find $\overline{E}$ at (-2, 2, 8) in air. **(05 Marks)**

c. State and prove the Gauss's law. **(06 Marks)**

d. Determine the volume charge density, if the field is $\overline{D} = \frac{10\cos\theta\sin\phi}{r}\hat{a}_r\ c/m^2$. **(04 Marks)**

2 a. Derive an equation for the potential at a point, due to an infinite line charge. **(06 Marks)**

b. If the potential field $V = 3x^2 + 3y^2 + 2z^3$ volts, find
i) V ii) $\overline{E}$ iii) $\overline{D}$ at P(-4, 5, 4) **(06 Marks)**

c. Deduce an equation for the capacitance of a coaxial cable of length 'L', radius of inner conductor 'a' and out conductor 'b'. **(08 Marks)**

3 a. State and prove the uniqueness theorem. **(06 Marks)**

b. Find the capacitance between the two concentric spheres of radii r = b and r = a, such that b > a, if the potential V = 0 at r = b, using the Laplace's equation. **(10 Marks)**

c. Determine whether or not the potential equations i) $V = 2x^2 - 4y^2 + z^2$ and ii) $V = r^2\cos\phi + \theta$ satisfy the Laplace's equation. **(04 Marks)**

4 a. State and prove the Stoke's theorem. **(04 Marks)**

b. If the magnetic field intensity in a region is $\overline{H} = (3y-2)\hat{a}_z + 2x\hat{a}_y$, find the current density at the origin. **(06 Marks)**

c. A co-axial cable with radius of inner conductor a, inner radius of outer conductor b and outer radius c carries a current I at inner conductor and –I in the outer conductor. Determine and sketch variation of $\overline{H}$ against r for i) r < a ii) a < r < b iii) b < r < c iv) r > c. **(10 Marks)**

PART – B

5 a. Derive an equation for the force between the two differential current elements. (06 Marks)

b. Derive the magnetic boundary conditions at the interface between the two different magnetic materials. Discuss the conditions. (08 Marks)

c. Calculate the inductance of a solenoid of 400 turns wound on a cylindrical tube of 10 cm diameter and 50 cm length. Assume the solenoid is in air. (06 Marks)

6 a. Using the Faraday's law, deduce the Maxwell's equation, to relate time varying electric and magnetic fields. (08 Marks)

b. Derive the Maxwell's equations in the point form of the Gauss's law for time varying fields. (06 Marks)

c. Given $\overline{E} = E_m \sin(\omega t - \beta z)\hat{a}_y$ in free space. Find $\overline{D}$, $\overline{B}$ and $\overline{H}$. (06 Marks)

7 a. Obtain the solution of wave equation for uniform plane wave in free space. (10 Marks)

b. State and explain the Poynting's theorem. (04 Marks)

c. For a wave traveling in air, the electric field is given by $\overline{E} = 6\cos(\omega t - \beta t)\hat{a}_z$ at f = 10 MHz. Calculate the average Poynting vector. (06 Marks)

8 a. Explain the reflection of uniform plane waves, with normal incidence at a plane dielectric boundary. (10 Marks)

b. Write short notes on:
   i) Standing wave ratio.
   ii) Skin effect in conductors.
(10 Marks)

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Third Semester B.E. Degree Examination, December 2010
Electronic Instrumentation

Time: 3 hrs.   Max. Marks:100

Note: ***Answer any FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Define accuracy, precision and significant figure. **(06 Marks)**
b. A true value of voltage across a resistor is 50V, the instrument reads 49V. Calculate the absolute error and percentage error. **(04 Marks)**
c. Explain with a circuit diagram, how a basic D' Arsonval movement can be converted into a DC voltmeter. Calculate a multiplier resistance required to measure voltage range of (0 - 25)V, when $R_M = 50$ ohms, $I_g = 50\ \mu A$. **(10 Marks)**

2 a. With a neat block diagram, explain the working principle of true RMS voltmeter. **(07 Marks)**
b. Write the advantages of a digital instrument over analog instruments. **(03 Marks)**
c. With a neat circuit diagram, explain the working of Ramp type DVM. **(10 Marks)**

3 a. With a neat block diagram, explain the working of digital frequency meter. **(07 Marks)**
b. Draw and explain the block diagram of CRT. **(07 Marks)**
c. Discuss the operation of electronic switch in oscilloscope. **(06 Marks)**

4 a. What is Barkhausen criterion? With a neat block diagram, explain the working of standard signal generator. **(10 Marks)**
b. What is sweep frequency generator? **(04 Marks)**
c. Draw and explain the block diagram of sampling oscilloscope. **(06 Marks)**

PART – B

5 a. What is a transducer? Explain the working of resistive position transducer. **(07 Marks)**
b. With a neat diagram, explain the principle and working of a LVDT. **(09 Marks)**
c. A resistance strain gauge with a gauge factor of 4 is connected to a steel member which is subjected to a strain of 1×10^{-6}. If the original gauge resistance is 150 ohms. Calculate the change in resistance. **(04 Marks)**

6 a. Derive an expression for capacitance comparison bridge at balance condition and calculate capacitance impedance at a frequency of 2 kHz, the bridge components are $C_3 = 100\ \mu F$, $R_3 = 100\ k\Omega$, and $R_2 = 50\ k\Omega$. **(09 Marks)**
b. Derive the bridge balance equation for the Kelvin's double bridge. **(07 Marks)**
c. Explain the sources and detectors in the bridge circuits. **(04 Marks)**

7 a. What is bolometer? Explain RF power measurement using bolometer bridge. **(07 Marks)**
b. Give the classification of digital displays. Compare the LEDs and LCDs. **(08 Marks)**
c. Explain in brief, the working of the photovoltaic transducer. **(05 Marks)**

8 Explain the following:
a. Piezo-electric transducer
b. Signal conditioning circuits
c. Photo transistor
d. Wagner earth connection
(20 Marks)

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MATDIP301

Third Semester B.E. Degree Examination, December 2010
Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Find the n^{th} derivative of $\log(ax + b)$. **(06 Marks)**

b. Find the n^{th} derivative of $\dfrac{x}{(1+3x+2x^2)}$. **(07 Marks)**

c. If $x = \sin t$ and $y = \text{cons}\ mt$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} + (m^2 - n^2)\, y_n = 0$. **(07 Marks)**

2 a. Show that the following pair of curves intersect each other orthogonally.
$r = a(1 + \sin\theta)$ and $r = a(1 - \sin\theta)$. **(06 Marks)**

b. Find the pedal equation of the curve $\dfrac{2a}{r} = 1 + \cos\theta$. **(07 Marks)**

c. Find the first five terms of the Maclaurin series of $f(x) = \log \sec x$. **(07 Marks)**

3 a. If $u = e^{ax - by} \sin(ax + by)$, show that $b\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y} = 2abu$. **(06 Marks)**

b. If $u = \sqrt{x^2 + y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, find $\dfrac{du}{dx}$ when $x = y = a$. **(07 Marks)**

c. If $z = f(x, y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that, $\left(\dfrac{\partial z}{\partial x}\right)^2 + \left(\dfrac{\partial z}{\partial y}\right)^2 = \left(\dfrac{\partial z}{\partial r}\right)^2 + \dfrac{1}{r^2}\left(\dfrac{\partial z}{\partial \theta}\right)^2$ **(07 Marks)**

4 a. Obtain the reduction formula for $\int \cos^n x\,dx$, where n is a positive integer. **(06 Marks)**

b. Show that $\displaystyle\int_0^{\pi} \frac{\sqrt{1-\cos\theta}}{1+\cos\theta} \sin^2\theta\, d\theta = \frac{8\sqrt{2}}{3}$. **(07 Marks)**

c. Evaluate $\displaystyle\int_0^{a}\int_0^{\sqrt{a^2-x^2}} x^2 y\, dy\, dx$. **(07 Marks)**

5 a. Prove that $\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$. **(06 Marks)**

b. Show that $\displaystyle\int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta \times \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi$. **(07 Marks)**

c. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$. **(07 Marks)**

6 a. Solve $(e^4 + 1)\cos x\, dx + e^4 \sin x\, dy = 0$. **(06 Marks)**

b. Solve $\left(x\tan \frac{y}{x} - y\sec^2 \frac{y}{x}\right)ds + x\sec^2\left(\frac{y}{x}\right)dy = 0$. **(07 Marks)**

c. Solve $(x + \tan y)\, dy = \sin 2y\, dx$. **(07 Marks)**

7 a. Solve $\frac{d^2y}{dx^2}+3\frac{dy}{dx}+2y=e^{-2x}$. **(06 Marks)**

b. Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=\cos 3x$. **(07 Marks)**

c. Solve $(D^2 - 5D + 1)y = 1 + x^2$. **(07 Marks)**

8 a. Prove that $(1 + \cos\theta + i\sin\theta)^n + (1+\cos\theta - i\sin\theta)^n = 2^{n+1}\cos^n\left(\frac{\theta}{2}\right)\cos\left(\frac{n\theta}{2}\right)$. **(06 Marks)**

b. Use Demoivre's theorem and solve the equation $x^4 - x^3 + x^2 + 1 = 0$. **(07 Marks)**

c. Expand $\cos^8\theta$ in a series of cosine of multiples of θ. **(07 Marks)**

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